Solutions to tutorial problems on Ch 15

1) The critical speed is given by $u_c = \beta\left[\lambda^2 + \mu^2 + \left(\frac{f_0}{2NH}\right)^2\right]^{-1}$.

As the phase-lines are N-S, $\mu = 0$.

$H = RT/g = 287 Jkg^{-1}K^{-1} \times 250K / 9.81 ms^{-2} = 7314m$

$N = 2\pi/\tau = 1.047 \times 10^{-2} s^{-1}$

$f_0 = 2 \times 7.292 \times 10^{-5} s^{-1} \sin 60^o = 1.263 \times 10^{-4} s^{-1}$

$\beta = \left(2 \times 7.292 \times 10^{-5} s^{-1} \cos 60^o\right) / \left(6.371 \times 10^6 m\right) = 1.145 \times 10^{-11} m^{-1} s^{-1}$

Hence $\left(\frac{f_0}{2NH}\right)^2 = 6.801 \times 10^{-13} m^{-2}$

We shall need λ, which is given by $\lambda = 2\pi/(20000km/n) = n\pi \times 10^{-7} m^{-1}$, where n is the number of waves round the latitude circle. Thus $\lambda^2 = n^2 9.870 \times 10^{-14} m^{-2}$

For completeness, computing this for the first 6 wavenumbers gives

n	1	2	3	4	5	6
$u_c / \left(ms^{-1}\right)$	14.7	10.7	7.30	5.07	3.64	2.70

2) Equation 10 can be re-written (using equation 11) as $\nu^2 = \frac{N^2}{f_0^2}\left[\frac{\beta}{\bar{u}} - \frac{\beta}{\bar{u}_c}\right]$. In the terms of the question this gives $\nu^2 = \frac{N^2\beta}{f_0^2\left(10.7ms^{-1}\right)}\left[\frac{1}{0.8} - \frac{1}{1}\right]$, so that $\nu = 0.429 \times 10^{-4} m^{-1}$.

This is the propagating case, so the amplitude goes as $\exp\frac{z^*}{2H}$.

> Over a distance of $40km$ the amplitude increases by a factor of $\exp\frac{4 \times 10^4 m}{2 \times 7314m} = 15.4$.

Over the same distance the phase changes by $0.429 \times 10^{-4} m^{-1} \times 4 \times 10^4 m = 1.716 radians$. As 2π *radians* corresponds to $10{,}000km$ for this latitude and wavenumber, we deduce that

> the wave troughs are $2730km$ further west at a height of $50km$ than they are at $10km$

3) This is the trapped case. The phase lines are vertical, so that part is trivial. The amplitude in this case goes as $\exp\left(\frac{1}{2H}-\nu_i\right)z^*$ and we have

$$\nu_i^{\,2} = \frac{N^2\beta}{f_0^2\left(10.7ms^{-1}\right)}\left[\frac{1}{1}-\frac{1}{1.2}\right]$$, so that $\nu_i = 3.50\times10^{-5}m^{-1}$ and

$$\left(\frac{1}{2H}-\nu_i\right) = 3.336\times10^{-5}m^{-1}.$$

Thus between the two heights the amplitude increases by a factor of $\exp\left(3.336\times10^{-5}m^{-1}\times4\times10^4 m\right) = 3.80$

4) TO APPEAR