Solutions: Chapter 12

1) Use subscript 1 to denote the first position and subscript 2 to denote the second position.

Then conservation of potential vorticity states $\frac{(\zeta_{abs})_1}{(\Delta p)_1} = \frac{(\zeta_{abs})_2}{(\Delta p)_2}$.

Now we have $(\zeta_{abs})_1 = f + (\zeta_{rel})_1 = 10^{-4}s^{-1} + 0$. Also $(\Delta p)_1 = 1000 - 200hPa$ and $(\Delta p)_2 = 700 - 200hPa$. This leads to

$$\boxed{(\zeta_{rel})_2 = -3.75 \times 10^{-5} s^{-1}}$$

note[1]

2) Proceed similarly to in question 1.
$(\zeta_{abs})_1 = f + (\zeta_{rel})_1 = 2\Omega \sin(60) + 0 = 1.261 \times 10^{-4} s^{-1}$.

$$\boxed{(\zeta_{rel})_2 = -4.73 \times 10^{-5} s^{-1}}$$

3) Proceed similarly to in question 1, but now $(\Delta p)_1 = 1000 - 500hPa$ and $(\Delta p)_2 = 700 - 400hPa$, giving

$$\boxed{(\zeta_{rel})_2 = -4 \times 10^{-5} s^{-1}}$$

4) Use subscript 1 to denote the first state and subscript 2 to denote the later state. Then the question gives $(\Delta p)_1 = 250hPa$ and $(\Delta p)_2 = 300hPa$, so that $(\zeta_{abs})_2 = 12 \times 10^{-5} s^{-1}$ and

$$\boxed{(\zeta_{rel})_2 = +2 \times 10^{-5} s^{-1}}.$$

Now we have seen that the relative vorticity is twice the rate of rotation, so in the terms of the question, the air rotates as if it were a solid disc rotating cyclonically (anticlockwise as this is the N. Hemisphere) about a vertical axis with an angular velocity of $+1 \times 10^{-5} s^{-1}$. Hence at a distance of 500km from the centre of the rotation, the windspeed is $500km \times 1 \times 10^{-5} s^{-1}$.

$$\boxed{\text{Velocity at edge of disc } = 5ms^{-1}}.$$

[1] Note that this uses the approximation that f is 10^{-4} s^{-1} at 45 N. Using the more accurate value $f=1.03 \times 10^{-4}$ s^{-1} gives the answer -3.86×10^{-4} s^{-1}

5) This question also depends on the idea that $\zeta_{abs} / \Delta p$ is conserved on fluid columns. As the column approaches the mountain from the west, $v = 0$ and $\frac{\partial u}{\partial y} = 0$, since everything is uniform in the y-direction. Thus $\zeta_{rel} = 0$ and $\zeta_{abs} = f$ for $x \leq -L$. From the equation for the thickness, we can see that at $x = -L$, $\Delta p = 500hPa$. Now over the mountain we still have $\frac{\partial u}{\partial y} = 0$, for the same reason as before. Thus over the mountain we have

$$f + \frac{\partial v}{\partial x} = \left[p_0 + (\delta p_0)\left[1 - \cos\left(x\pi/L\right)\right]\right] \times f / (500hPa)$$

or, on re-arranging and substituting the given values,

$$\frac{\partial v}{\partial x} = \frac{f}{20}\left(-1 - \cos\frac{\pi x}{L}\right).$$

Integrating this w.r.t. x from $-L$ to $+L$ gives

$$\boxed{v = -\frac{f2L}{20} = -\frac{10^{-4}s^{-1} \times 2 \times 500km}{20} = -5ms^{-1}}$$