Atmospheric Dynamics Lect.12: Potential Vorticity

Stephan Matthiesen

Institute for Atmospheric and Environmental Sciences

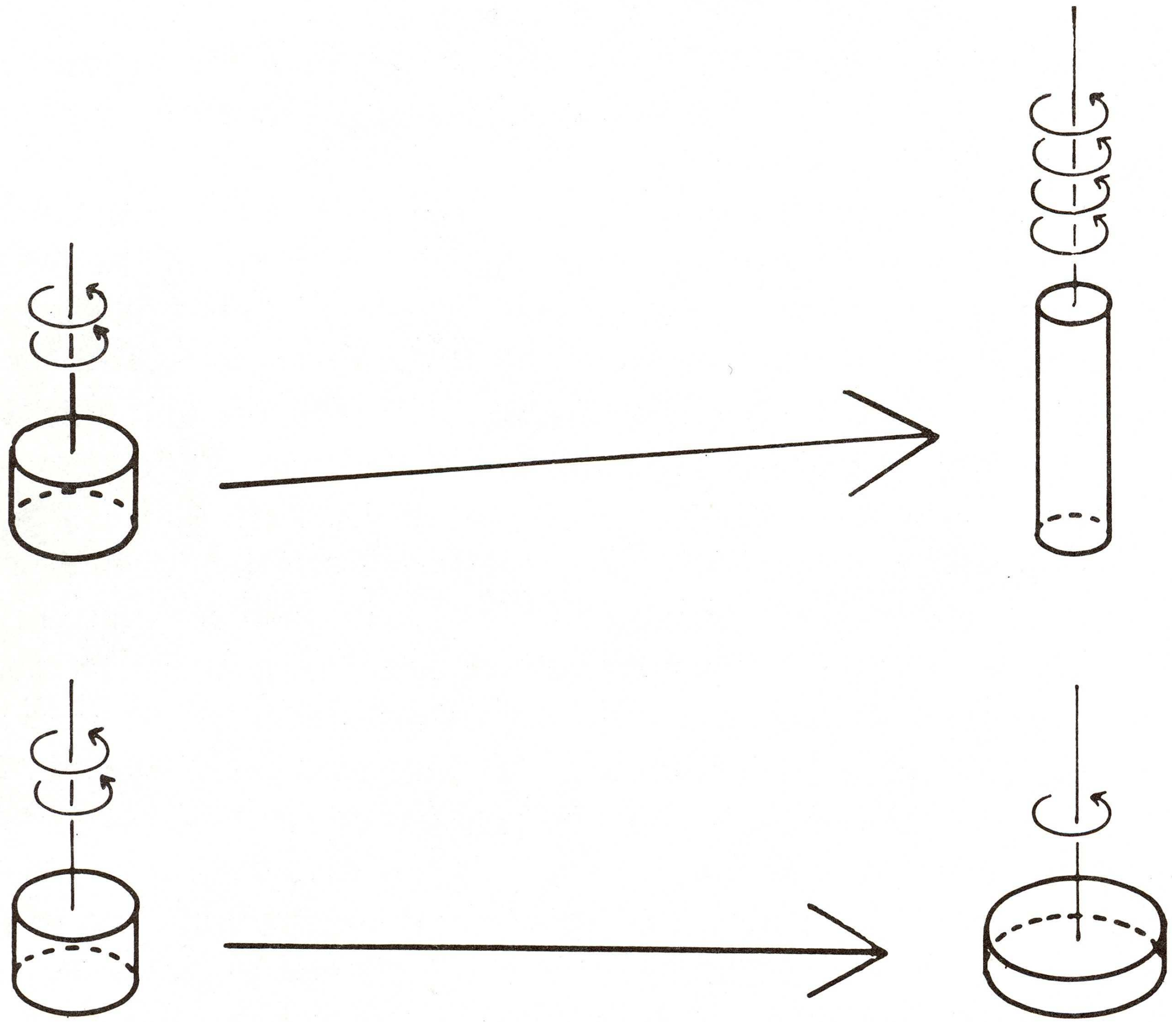
School of Geosciences, Edinburgh University

Crew Building room 218

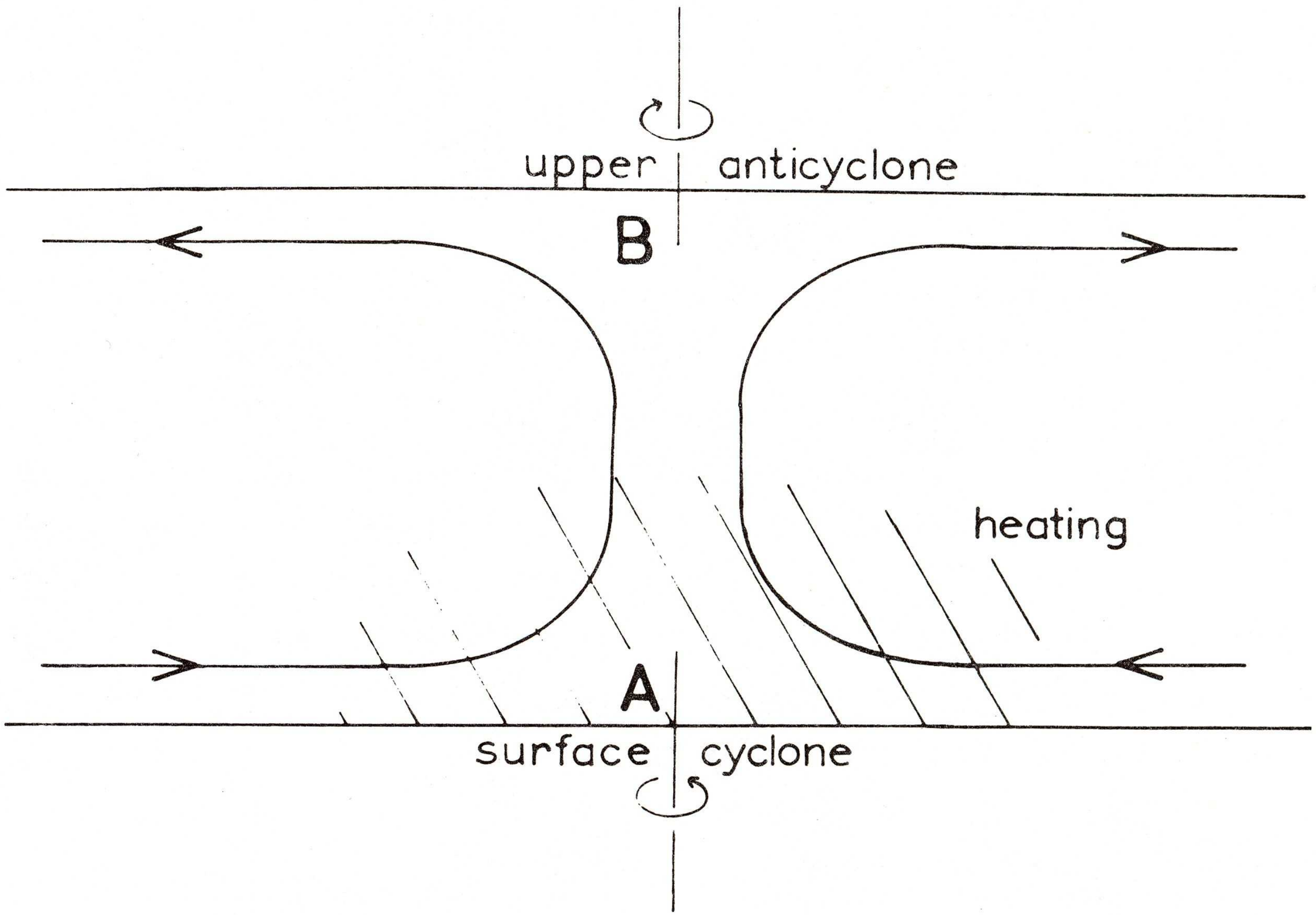
Scope of Lecture

- The heat low
- Limits to vorticity values
- Deflection of flow by a mountain
- Potential vorticity of air column
- Potential vorticity at a point

Vorticity changes



Heat low (mechanism)



Heat low (example)

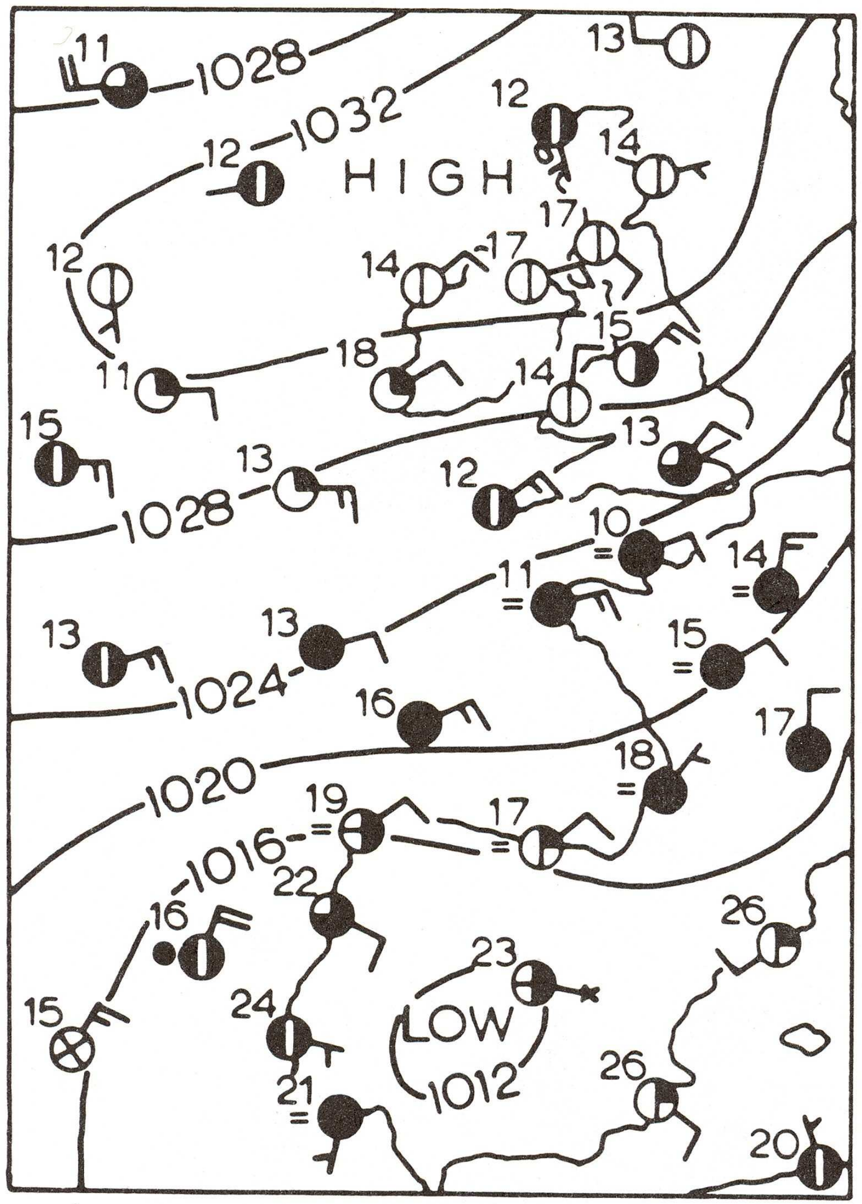


Fig. 4.15 A heat low over Spain. From *Daily Weather Report* 1200 GMT, 29 May 1963.

Limit to vorticity

The vorticity equation is

$$\left(\frac{D}{Dt}\right)_p \zeta_{abs} = -\zeta_{abs} div_p \mathbf{v}$$

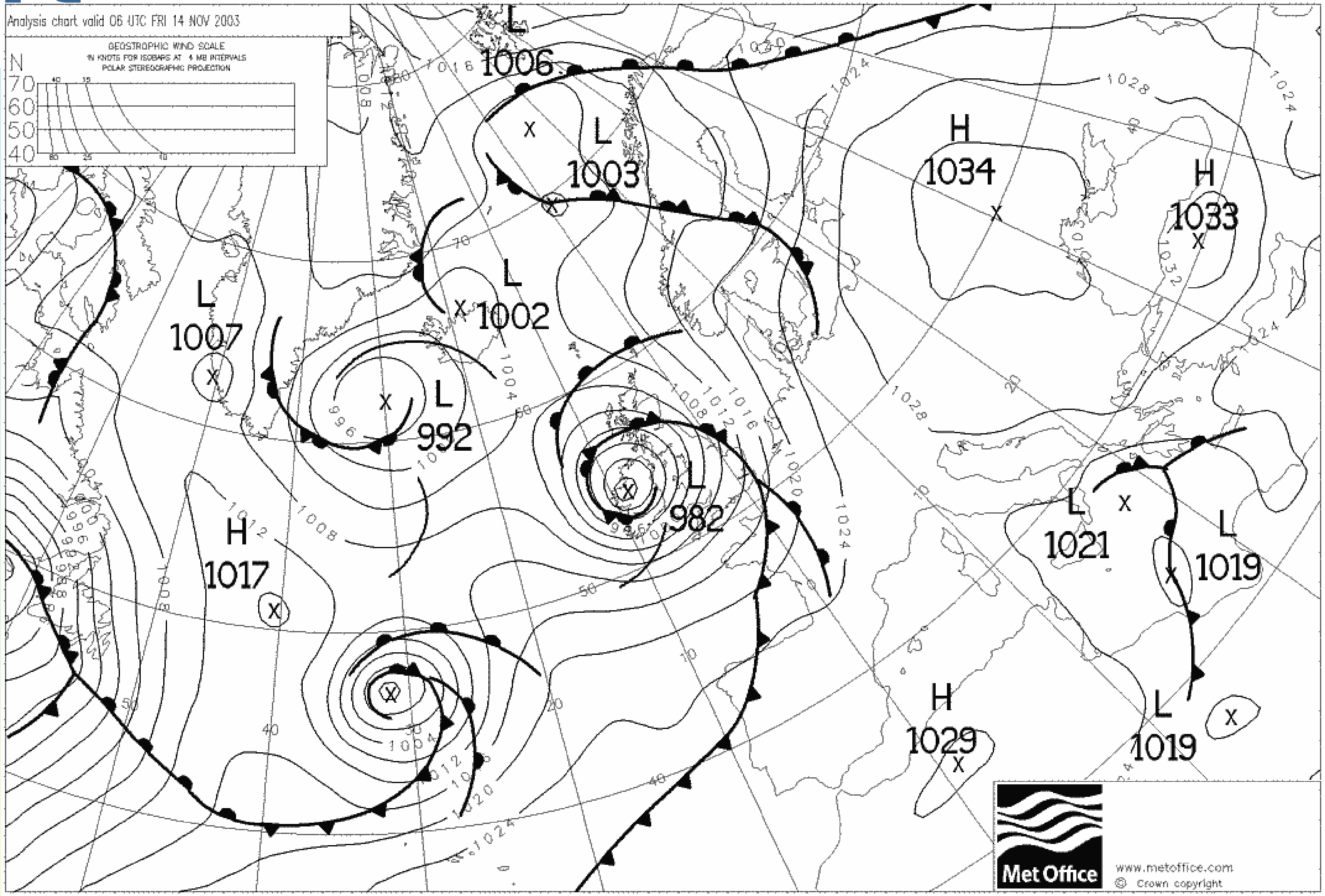
$$\ln\frac{(\zeta_{abs})_2}{(\zeta_{abs})_1} = \int_{t_1}^{t_2} \frac{1}{\zeta_{abs}} \frac{D\zeta_{abs}}{Dt} dt = \int_{t_1}^{t_2} div_p \mathbf{v} dt$$

Thus sign of absolute vorticity is unaltered

Thus it is always positive

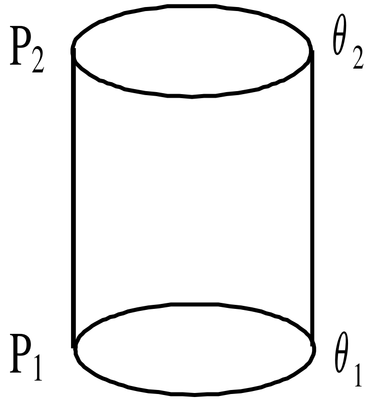
$$-f \le \zeta_{rel} < +\infty$$

Surface weather chart



Cyclones more vigorous than anticyclones

Potential vorticity



$$\left(\frac{D}{Dt}\right)_p (\zeta_{abs}) = \zeta_{abs} \frac{\partial \varpi}{\partial p}$$

$$\frac{1}{\delta p} \frac{D\delta p}{Dt} = \frac{\partial \varpi}{\partial p}$$

$$\delta p = p_1 - p_2$$

$$\left(\frac{D}{Dt}\right)_p (\zeta_{abs}) - \frac{\zeta_{abs}}{\delta p} \frac{D\delta p}{Dt} = 0$$

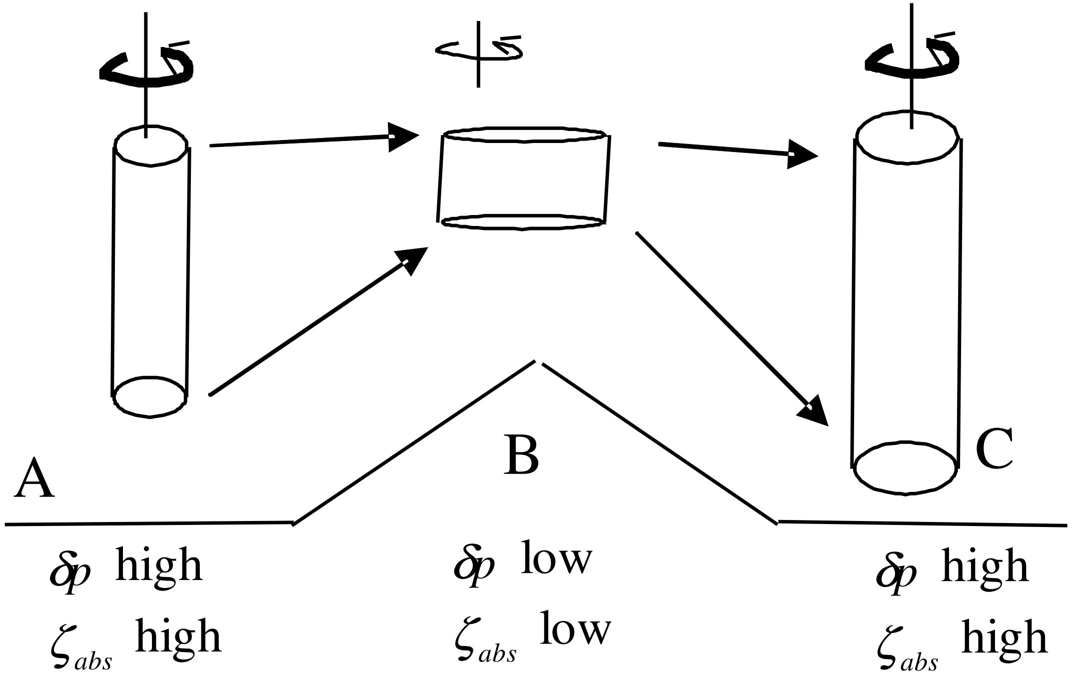
$$\left(\frac{D}{Dt}\right)_p \frac{\zeta_{abs}}{\delta p} = 0$$

$$\frac{\zeta_{abs}}{\delta p}$$

a.k.a. the potential vorticity is conserved

In frictionless flow

Flow over mountain



At A v=0

$$\frac{\partial u}{\partial y} = 0$$

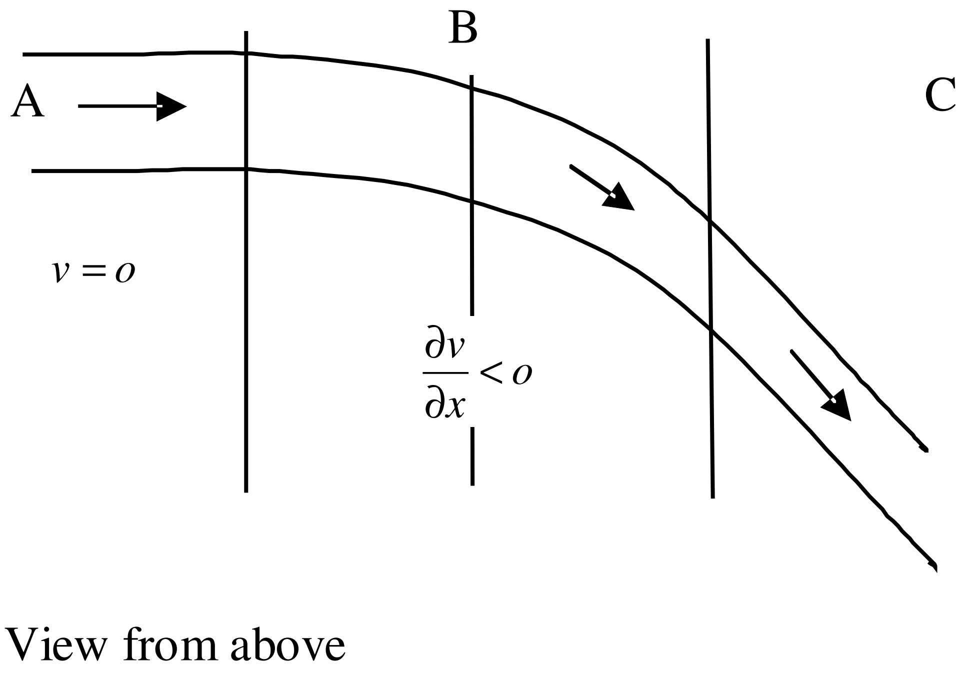
$$\zeta_{rel} = 0$$

$$\zeta_{abs} = f$$

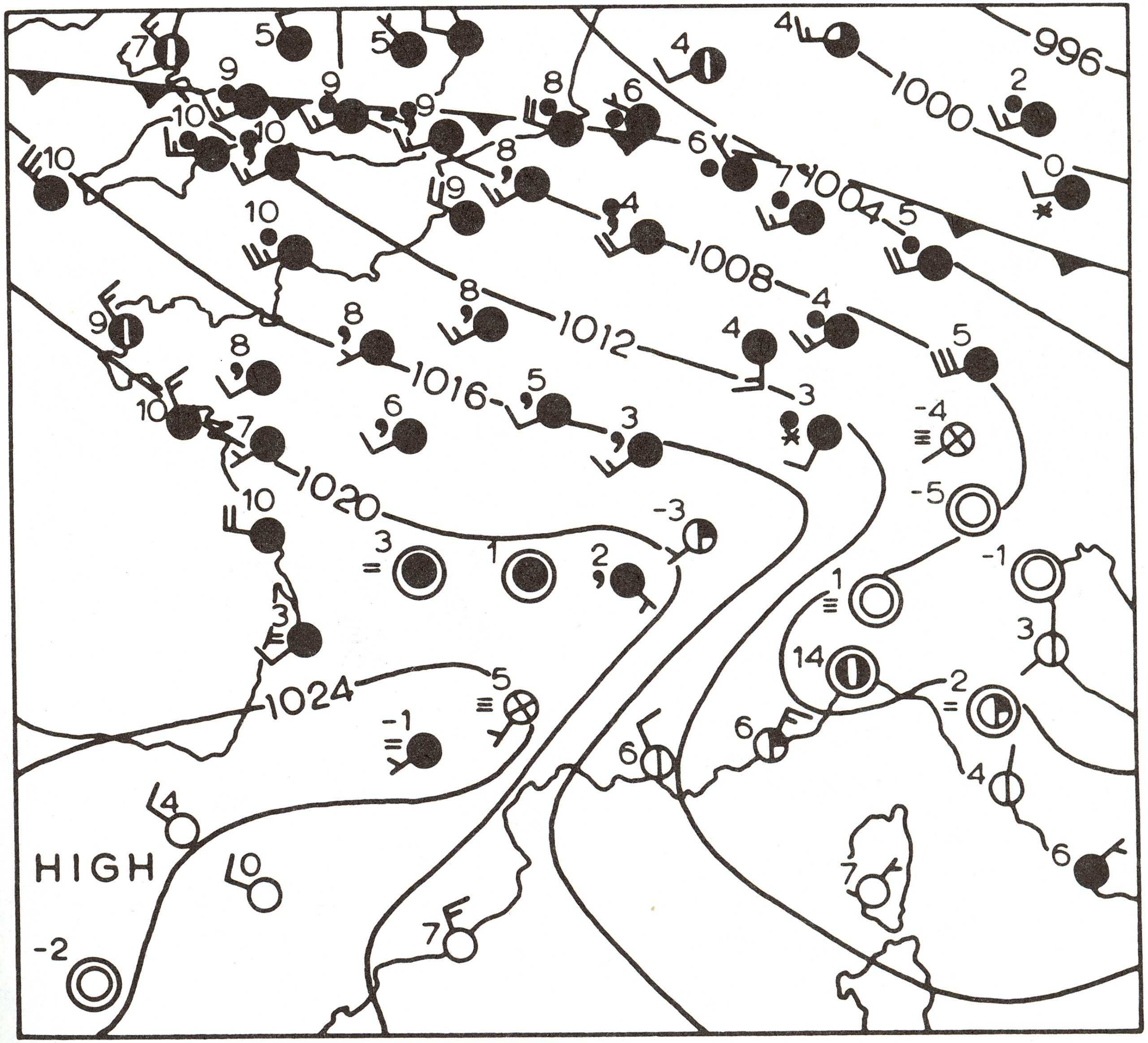
At B v=0 δp small

$$\zeta_{abs} < f \qquad \zeta_{rel} < 0 \qquad \frac{\partial v}{\partial x} < 0$$

Flow over mountain (cont.)

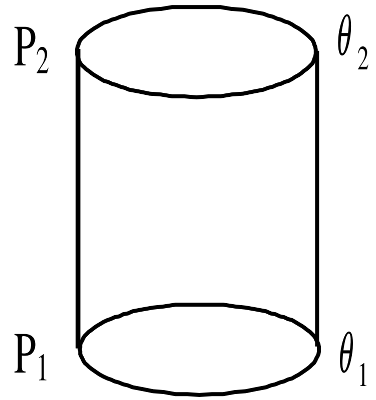


Mountain deflection (example)



Potential vorticity at a point

$$\frac{1}{p_1 - p_2}\zeta_{abs}$$



$$\frac{\theta_1 - \theta_2}{p_1 - p_2}\zeta_{abs}$$

$$\frac{\partial \theta}{\partial p}\zeta_{abs} \qquad \frac{1}{\rho}\frac{\partial \theta}{\partial z}\zeta_{abs}$$

Ertel's or isentropic potential vorticity

Full 3-d version: $$Z = \frac{1}{\rho}\left(\boldsymbol{\zeta}_{\mathbf{rel}} + \boldsymbol{\Omega}\right)\nabla\theta$$

when the flow is strongly stable $\frac{\partial\theta}{\partial z}$ large

=> gradient of pot. Temperature close to vertical

$$Z = \frac{1}{\rho}\left(\zeta_{\mathrm{rel}} + f\right)\frac{\partial\theta}{\partial z}$$