Atmospheric Dynamics

Solutions to Tutorial Questions on Chapter 8

1) You can answer this question is several ways. Only one is sketched out here. Thus your answer may differ from this, but the ball park answer should be similar.

Let values at the surface have subscript o and those at a height of 1 km have subscript 1. We are invited to estimate $\frac{\rho_0 - \rho_1}{\rho_0}$. Let us first do it for an isothermal atmosphere. The gas law then tells us that $\frac{\rho_o - \rho_1}{\rho_0} = \frac{p_0 - p_1}{p_0}$. The hydrostatic equation in integrated form gives us $p_1 = p_0 \exp - \frac{g}{RT}(z_1 - z_0)$. Choosing $T = 288K$ and $p_0 = 1000hPa$ gives $p_1 = 888hPa$, so that $\frac{\rho_o - \rho_1}{\rho_0} = \frac{888 - 1000}{1000} = -0.11$ or a decrease of 11%.

This suggests that we will get substantially similar estimates for non-isothermal estimates. For instance in atmospheres for which the temperature decreases with height the temperature will only vary by 10K in a kilometre, which is a fractional change in temperature of around 3 percent. Hence our estimate of the fractional changes in density remain correct to the quoted two decimal places.

2) From the notes the ageostrophic wind is given by $U = B\{\exp - i\alpha z\}\exp - \alpha z$, with $\alpha \equiv \sqrt{\frac{f}{2K}}$. Now as a result of the no-slip boundary condition U at the surface is exactly opposite in direction to the geostrophic wind. So it is next opposite in direction to the geostrophic wind when it has turned through an angle of 2π. The first exponential controls the rotation, so the height we seek satisfies $\alpha z = 2\pi$ or

$$z = 2\pi\sqrt{\frac{2K}{f}}$$

At 60°N $f = 2\Omega \sin\phi = 1.263 \times 10^{-4} s^{-1}$, giving $z = 2.09km$

3) Magnitude of the ageostrophic wind $\propto \exp - \alpha z$. This falls by a factor 10 when αz increases by 2.3 (this factor 2.3 is worth remembering). This gives

$$z = 727m$$

4) The frictional stress is in the opposite direction to the wind shear, i.e. to $\frac{dU}{dz}$. This means that the friction is parallel to the tangent to the spiral. We argue in the notes

that the wind just above the surface is parallel to that windshear, so the frictional force on the air at the surface is in the opposite direction to the wind just above the surface.