Atmospheric Dynamics

Solutions to Tutorial Questions on chapter 7

1) By definition $\bar{s} = \frac{1}{2\tau}\int_{t_o-\tau}^{t_o+\tau} s.dt$ and $s' = s - \bar{s}$.

Thus

$$\overline{(s')} = \frac{1}{2\tau}\int_{t_o-\tau}^{t_o+\tau}(s-\bar{s}).dt = \frac{1}{2\tau}\int_{t_o-\tau}^{t_o+\tau}(s).dt - \frac{1}{2\tau}\int_{t_o-\tau}^{t_o+\tau}(\bar{s}).dt$$
$$= \bar{s} - \frac{\bar{s}}{2\tau}\int_{t_o-\tau}^{t_o+\tau} dt = \bar{s} - \bar{s} = 0$$

QED

It is obvious that $\bar{\bar{s}} = \bar{s}$, since $\bar{s}$ is not a function of t in the terms of the question (i.e. time changes in the average quantities negligible.)

$$\overline{sr} = \overline{(\bar{s}+s')(\bar{r}+r')} = \overline{(\bar{s}\bar{r}+\bar{s}r'+s'\bar{r}+s'r')} = \left(\overline{\bar{s}\bar{r}}+\overline{\bar{s}r'}+\overline{s'\bar{r}}+\overline{s'r'}\right)$$
$$= \bar{s}\bar{r} + \bar{s}\overline{r'} + \overline{s'}\bar{r} + \overline{s'r'} = \bar{s}\bar{r} + \overline{s'r'}$$

QED

2) $\bar{u} = 4u_o$, and $\bar{w} = w_o$.

So that the flux due to steady flow = $\rho\bar{u}.\bar{w}. = \rho 4u_o w_o = 5kgm^{-1}s^{-2}$.

$u' = u_o\cos(ht+\varepsilon)$ and $w' = w_o\cos(ht)$. Hence

$$u'w' = u_o w_o \cos(ht+\varepsilon)\cos(ht)$$
$$= u_o w_o\left(\frac{1}{2}\right)[\cos(2ht+\varepsilon)+\cos(\varepsilon)]$$

When the time varying term is averaged over many periods the average tends to zero (exactly zero if the averaging interval is exactly the period of course). (Note that the "many" is implied by our requirement that the time-mean varies only slowly). Thus

$$\rho\overline{u'w'} = \rho u_o w_o \frac{1}{4\tau}\int_{t-\tau}^{t+\tau}(\cos\varepsilon)dt = \frac{\rho u_o w_o}{2}\cos\varepsilon$$

Now $\frac{\rho u_o w_o}{2} = 0.9kgm^{-1}s^{-2} = 0.9Nm^{-2}$, so we obtain the following values

a) 0.9 Nm^{-2}, b) 0 c) -0.9 Nm^{-2} d) 0

3) The information in the question allows us to form an estimate of $\frac{\partial \overline{u}}{\partial z}$, namely $\frac{\partial \overline{u}}{\partial z} = \frac{(10-8)ms^{-1}}{1m} = 2s^{-1}$.

Thus $\tau_{xz} = -1.2kgm^{-3}4m^2s^{-1}2s^{-1} = -9.6Nm^{-2}$