Solutions to problems on Chapter 6

1) A typical windspeed is $10ms^{-1}$, and we can use the geostrophic equation in p- co-ordinates to get the slope of the isobaric surface. This gives the slope as $\frac{v_g f}{g}$. Putting in typical values gives a slope of about 1 in 10,000.

2) The main assumption will be that the wind is approximately geostrophic. This will allow us to use the idea of the previous question, giving $\frac{\partial z}{\partial y} = -\frac{fu}{g}$.

It is possible to integrate this assuming f is constant (say at the value for 45N).

A better approximation will be to note that $dy = ad\phi$, where a is the radius of the earth and ϕ is latitude. Thus $\frac{dz}{d\phi} = -\frac{2ua\Omega \sin\phi}{g}$.

Integrating this with respect to latitude gives

$$z_2 - z_1 = \left(\frac{2au\Omega}{g}\right)[\cos\phi_2 - \cos\phi_1]$$

and putting in values gives **height difference = 69.4 m**

3) The lines of constant thickness are readily found to run from NE to SW with low thickness to the SE, and with spacing of $200km/\sqrt{2}$ between contours at $10m$ spacing.

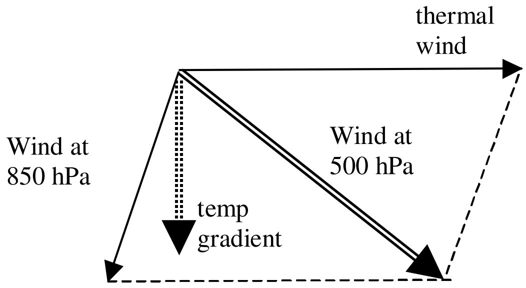
The magnitude of the thermal wind v_T is given by $\frac{g}{f}|\nabla z'|$ where z' is the thickness. Using the values just computed gives

$$\boxed{v_T = 7.0ms^{-1}}.$$

From the vertically integrated form of the hydrostatic equation it is easily shown that $\delta z' = \frac{R}{g}\ln\left(\frac{p_1}{p_2}\right) \times \delta\overline{T}$, where $\delta z'$ is a change in thickness and $\delta\overline{T}$ is a change in the average temperature. We have just found the gradient in thickness, hence we can obtain that

the gradient in average temperature is $5.9K/(1000km)$.

4) The sketch shows the orientation of the wind at 850hPa and the temperature gradient as given in the question. From this we can deduce that the thermal wind is directed to towards the east. The wind at the higher level will be the resultant of the lower level wind and the thermal wind. (All this assumes that the geostrophic wind is a reasonable approximation to the true wind.)



Use $\mathbf{v}_T = \frac{R}{f}\ln\left(\frac{p_1}{p_2}\right)\mathbf{k}\wedge\nabla\overline{T}$ to get that the magnitude of the thermal wind is 60.9 ms^{-1}

Applying the cosine rule gives that the

magnitude of the wind at 500hPa is $51.4ms^{-1}$.

The direction is obtained from the sine law. This gives that the

wind blows from a bearing of 348 deg.

(Winds are reported in terms of the direction they are blowing from, with the angle measured clockwise from North. This means that a wind from W to E has a bearing of 270 deg, while one from NW to SE has a bearing of 315 deg.)

5) The distance from the pole to the equator is 10,000 km (The meter was originally defined so as to make this exact - but this was found to be a rather impractical definition) so the temperature gradient is $(40K/10^7 m)$. Applying $\mathbf{v}_T = \frac{R}{f}\ln\left(\frac{p_1}{p_2}\right)\mathbf{k}\wedge\nabla\overline{T}$ with the Coriolis parameter for 45°N gives a thermal wind of 18.5 ms^{-1}, while that for 60°N 14.6 ms^{-1}.

Thus the wind at 200 hPa is about $15\,ms^{-1}$ more westerly.

6) More or less self-evident. A good way to approach the problem is to work out the sign of the component of the temperature gradient in the direction of the surface wind. This in turn will depend on the sign of the component of the thermal wind perpendicular to the surface wind. The argument is completed by deciding how this is related to the change in wind direction with increasing height.