Atmospheric Dynamics

Solutions to exercises on Lect 5.

1) Applying the equation for the geostrophic wind, with the magnitudes in the question gives a geostrophic wind of 33.3 ms^{-1} from the SW.

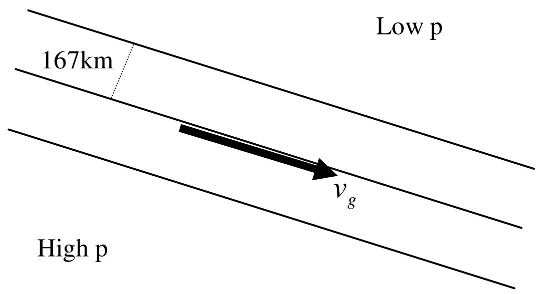
2) a) 23.9 ms^{-1} from the SW
b) 68.9 ms^{-1} from the SW
c) 135 ms^{-1} from the SW
d) 47.1 ms^{-1} from the NE
e) 27.2 ms^{-1} from the NE

3) Use the pressure and temperature in the gas equation to work out the new values of density, then use those values of density in the geostrophic equation.

The respective densities work out as 1.38, 1.28 and 1.13 kg m^{-3}, and the geostrophic windspeeds as 29.0 m s^{-1}, 31.3 m s^{-1} and 35.4 m s^{-1} all from the SW.

Note that the change in temperature (and hence density) does produce significant changes to the magnitude, but not such major changes as is produced by the variations with latitude.

4) The configuration of isobars is as in the sketch. To a first approximation, the geostrophic wind is an approximation to the true wind. The magnitude of the geostrophic wind is given by



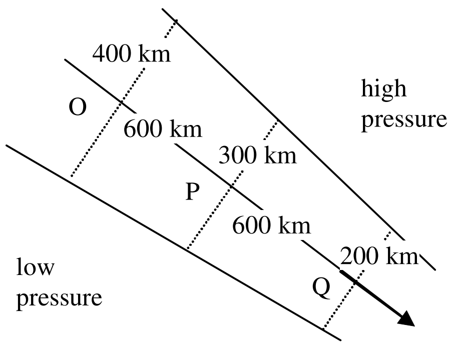
$$\frac{1}{\rho f}|\nabla p| = \frac{400Pa}{1.2kgm^{-3}10^{-4}s^{-1}167\times10^{3}m}$$

$$\boxed{=20\ \text{ms}^{-1}}.$$

The wind is along the isobars with low pressure on the left, thus the **<u>wind blows from the WNW</u>** in the case of parallel isobars.

Turning now to the situation where the isobars are not parallel, but instead are getting closer together. This means that the geostrophic wind is accelerating towards the ESE, and so the real wind must have the same acceleration (to order of the Rossby number). The acceleration is the resultant of the specific Coriolis force and the specific pressure gradient force. As the s.p.g.f. is perpendicular to the isobars and we have just deduced that the acceleration has a component parallel to them (in the direction towards ESE), the s.C.f. must have such a component. As the s.C.f. is perpendicular to the wind and to the right of it, the wind must have a component towards low pressure in this case. That is, the wind is rotated anticlockwise from the first estimate.

5) The situation described looks as in the sketch



The arrow shows the direction of the geostrophic wind.

A possible estimate of the actual wind at P is the geostrophic wind. Using the geostrophic wind equation shows this to be $11.1ms^{-1}$ towards the SE.

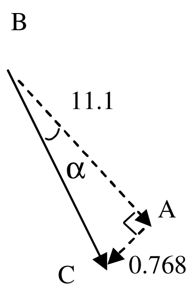
We are however asked for our best estimate of the wind. We can improve our estimate by noting that in the situation drawn, the air is accelerating, and we can estimate that acceleration by applying the geostrophic equation to the terms in the acceleration and thereby estimate the ageostrophic component. Adding this to the geostrophic component will then give an improved estimate.

(An alternative way of thinking about this is to say that the acceleration is the resultant of the specific pressure gradient force and the specific Coriolis force. We can estimate the acceleration, we know the pressure gradient, so we can estimate the Coriolis force on the true wind, and hence the wind which produces it. That wind will be our better estimate.)

Applying the geostrophic equation gives magnitudes of the geostrophic wind at O of $8.3ms^{-1}$ towards the SE and at Q of $16.6ms^{-1}$ towards the SE. Hence, as the situation is steady, we can estimate the acceleration as $\mathbf{v}.\nabla\mathbf{v}$, viz,

$$\left(11.1ms^{-1}\right)\times\left(\frac{16.6ms^{-1}-8.3ms^{-1}}{1200km}\right) \text{ towards the SE } = +7.68\times10^{-5}ms^{-2} \text{ to the SE.}$$

Since $\mathbf{v}_a = \frac{1}{f}\mathbf{k}\wedge\frac{D\mathbf{v}}{Dt}$, and f is negative, $\mathbf{v}_a$ is towards the SW and has value $= +7.68\times10^{-1}ms^{-1}$



In the sketch, BC is the best estimate of the wind. AC is the ageostrophic wind and BA is the geostrophic wind. (Note that BA is perpendicular to AC in the problem, but is generally not so.)

The magnitude of AB is

$\sqrt{11.1^2+0.768^2} = 11.1\ ms^{-1}$.

Angle α is readily found to be 3.9 degrees

6) This is a simple application of the gradient wind equation. In the cyclonic case we have $fV_g - fV = V^2/R$ with $V = 15ms^{-1}$, $f = 10^{-4}s^{-1}$ and $R = 500km$. This gives

$$\boxed{V_g = 19.5ms^{-1}}$$

For the anticyclonic case, using the appropriate signs in the equation gives

$$\boxed{V_g = 10.5ms^{-1}}$$

7) This again requires the gradient wind equation, but this time we are finding V.

$$\boxed{\text{Cyclonic case } V = 7.02ms^{-1}}$$

$$\boxed{\text{Anticyclonic case } V = 10ms^{-1}}$$

8) In a steady circular anticyclone we must have $V_g \le \frac{Rf}{4}$ for a physical solution to be possible. We can write $V_g = -\frac{1}{\rho f}\frac{\partial p}{\partial R}$. (I have chosen a sign convention that the geostrophic wind is positive, but of course the pressure decreases outwards in an anticyclone.)

Hence in the limit we have $\frac{\partial p}{\partial R} = -\left(\frac{\rho f^2}{4}\right)R$, leading to

$$p = p_c - \left(\frac{\rho f^2}{8}\right)R^2$$

where p_c is the pressure at the centre. Substituting the values given in the question, we find

$$\boxed{p_c = 1015hPa}.$$