Atmospheric Dynamics

Solutions to Tutorial Questions on Chapters 3

1) By the terms of the question it is clear that $\nabla s = \left(\frac{\partial s}{\partial x}, \frac{\partial s}{\partial y}, ? \right) = \left(141 pbbv / 10^5 m, 0, ?)\right)$ and $v = \left(-5.\cos 45 ms^{-1}, -5.\cos 45 ms^{-1}, 0\right)$, so that

$$\frac{\partial s}{\partial t} = -\mathbf{v}.\nabla s = \frac{5 \times 141}{\sqrt{2} \times 10^5} (ppbv) s^{-1} = 5 \times 10^{-3} (ppbv) s^{-1}.$$

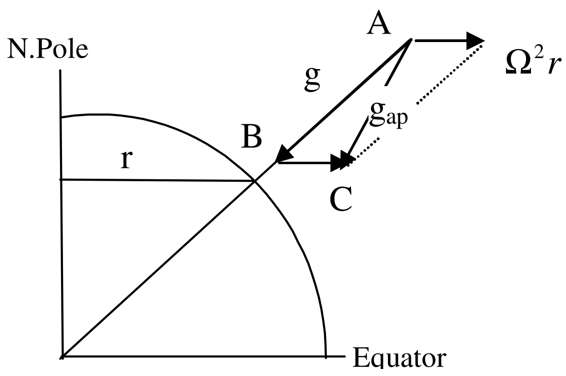
In 1 hour the mixing ratio will increase by $3600 \times 5 \times 10^{-3}\, ppbv$, giving a new mixing ratio of $328 ppbv$.

2) The difference between true and apparent gravity is due to the centrifugal force.

a) At the pole, there is no centrifugal force, so the difference is zero

b) At the equator, $\Omega^2 R = \left(7.292 \times 10^{-5} s^{-1}\right)^2 \times \left(3671 \times 10^3 m\right) = 3.39 \times 10^{-2} ms^{-2}$, and the centrifugal force is directed away from the axis of rotation, i.e. in the opposite direction to gravity. Thus apparent gravity is less than true gravity by $3.39 \times 10^{-2} ms^{-2}$

At 45 N we have a configuration shown in the sketch. Now



$$BC = \Omega^2 r = \Omega^2 R \cos 45$$
$$= 3.39 \times 10^{-2} \cos 45 ms^{-2}$$

AC = g_{ap}

AB = g

To get magnitude of g_{ap} use the cosine rule

$$AC^2 = BC^2 + BA^2 - 2BC.BA \cos ABC$$

This allows us to find AC, giving

$$g_{ap} = g\sqrt{1 + \frac{\left(3.39 \times 10^{-2} \cos 45\right)^2}{g^2} - \frac{2 \times 3.39 \times 10^{-2} \cos 45}{g}}$$

Ignore second term under square root in comparison with third and use binomial theorem to expand the root gives

$$\begin{aligned} g_{ap} &= g - \frac{3.39\times10^{-2}\cos 45}{g} \\ &= \left(9.81 - 2.44\times10^{-3}\right) ms^{2} \end{aligned}$$

To get the angle BAC, use the sine rule

$$\frac{\sin A}{BC} = \frac{\sin 45}{AC}$$

so that

$$\sin A = \frac{3.39\times10^{-2}\cos 45 \sin 45}{9.81} = 1.73\times10^{-3}$$

$$A = 9.85\times10^{-2} \text{ degrees}$$