Atmospheric Dynamics

Tutorial 2: Solutions.

1) The hydrostatic equation is $\frac{dp}{dz} = -\rho g = -\frac{pg}{RT}$ on using the gas law.

Over 25m an acceptable approximation is $\delta p = -\frac{pg}{RT}\delta z$.

Substituting the values gives $\delta p = -292Pa = 2.9hPa$

2) Use $\ln\left(\frac{p_2}{p_1}\right) = -\int_{z_1}^{z_2} \frac{g}{RT} dz = -\frac{g}{RT}(z_2 - z_1)$, on integration for an isothermal atmosphere. Make temperature the subject of the formula and substitute values to give

$$T = 246K$$

3) Make use of $\theta = T\left(\frac{p_0}{p}\right)^\kappa$ and $\mathrm{T} = \theta\left(\frac{\mathrm{p}}{\mathrm{p}_0}\right)^\kappa$, giving

pressure/hPa	Temperature	Potential Temp/ oK
500	-20 ^{o}C	**308**
10	**228K or -49^{o}C**	850
800	30 ^{o}C	**323**
200	**221K or -52^{o}C**	350
88.4	230 ^{o}K	460

4) $\theta = T\left(\frac{p_0}{p}\right)^\kappa = 316K = 42.7C$ on substituting appropriate values. So on bringing the particle adiabatically to 1000hPa, its temperature would be 42.7C.

To find the temperature at 200hPa use $\mathrm{T} = \theta\left(\frac{\mathrm{p}}{\mathrm{p}_0}\right)^\kappa$ with appropriate values. Gives T=200K=-73C

5) $\frac{d\theta}{dz} = \frac{d}{dz}\left(T\left(\frac{p_o}{p}\right)^\kappa\right) = T\frac{d}{dz}\left(\left(\frac{p_o}{p}\right)^\kappa\right)$ (using isothermal property)

$= Tp_o{}^\kappa(-\kappa)p^{-\kappa-1}\frac{dp}{dz} = Tp_o{}^\kappa(-\kappa)p^{-\kappa-1}(-\rho g)$ (by hydrostatic equation)

Now use gas law, leading to

$$\frac{d\theta}{dz} = \left(\frac{p_o}{p}\right)^{\kappa} \frac{\kappa g}{R}$$

There are various ways of rewriting this equation using relationships between κ γ, C_p, C_v and R. A neat version is

$$\boxed{\frac{d\theta}{dz} = \left(\frac{p_o}{p}\right)^{\kappa} \frac{g}{C_p}}$$

Substituting $p = p_0$ gives $\boxed{\frac{d\theta}{dz} = 9.8Kkm^{-1}}$.

6) We have $N^2 = \frac{g}{\theta}\frac{d\theta}{dz}$. Use the result of the previous question and the definition of potential temperature to give $N^2 = \frac{g^2}{C_p T}$.

The question does not specify a temperature to use. If we take 250K as a temperature of the middle troposphere, substitution of the values of the constants gives $N = 1.96 \times 10^{-2} s^{-1}$. The corresponding period is $2\pi / N = 321s$ =5mins 21 secs.

7) The Brunt-Vaisala frequency will be imaginary when $\frac{d\theta}{dz}$ is negative, i.e. when potential temperature decreases with height.

In the notes it was shown that the displacement δz of an air parcel from its initial position has solutions proportional to $\exp(\pm iNt)$. If N is imaginary we could write $N = -i|N|$. Thus when N is imaginary, the displacement has solutions proportional to $\exp(\pm i^2 |N| t) = \exp(\pm |N| t)$. Thus one of these grows exponentially. The total solution will be a linear sum of these two components, and the exponentially growing one will always eventually dominate. Hence and displacement given to an air parcel in an atmosphere in which the potential temperature decreases upwards will grow exponentially. This situation is therefore unstable.