Atmospheric Dynamics

Solutions to Tutorial 1:

1) Volume mixing ratio is equivalent to the ratio of the number of molecules of each gas present. However 1 mole of any gas contains the same number of molecules as a mole of any other gas, so the volume mixing ratio is also the ratio of the number of moles of each gas present.

2) This comes straight from the table of constituents as 9.3×10^{-3}.

3) We have the gas law $p = R\rho T$. The question gives us the pressure, so a guess at a suitable temperature leads to the corresponding density. Setting the temperature as 20 C (293K) gives

$$\rho = 1.19\text{kgm}^{-3}$$

4) $n = \sum_i n_i$ (i.e. the total number of molecules in the mixture is the some of the numbers in each constituent).

But $n = M / \overline{m}$ and $n_i = M_i / m_i$ (i.e. the number of molecules in each case is the total mass of constituent divided by the mass of each molecule for that constituent – with a similar statement for the total mixture)

Hence the result.

5) From the gas law, we see that the density of gases at fixed pressure and temperatures is inversely proportional to the gas constant and hence proportional to the apparent molecular weight of the gas. As the molecular weight of water (2+16=18 kg/kmole) is less than that of dry air (28.96 kg/kmole), the molecular weight of moist air is less than that of dry air.

Hence the density of moist air is less than that of dry air at the same temperature and pressure.

6) $$m.m.r. = \frac{mass\_of\_constitutent}{mass\_of\_dry\_air}$$
$$= \frac{(no\_of\_molecules\_of\_constituent)\times(mol\_wt\_of\_constituent)}{(no\_of\_molecules\_of\_dry\_air)\times(mol\_wt\_of\_dry\_air)}$$
$$= v.m.r.\times\frac{mol\_wt\_of\_constituent}{mol\_wt\_of\_dry\_air}$$

7) Let $\hat{T}$ be the virtual temperature of the sample of moist air, and let m_m be the molecular weight of the moist sample and let its pressure, density and true

temperature be p, ρ and T as in the standard notation. Using the gas law for the moist sample we have $p = \frac{R}{m_m}\rho T$.

By the definition of $\hat{T}$ we must have $p = \frac{R}{m_d}\rho\hat{T}$ (since that is what the gas law would look like for dry air with the same density and pressure).

It follows that $\hat{T} = \frac{m_d}{m_m}T$.

Now if r is the mass mixing ratio of water in the moist sample and M_d and M_v are respectively the masses of dry air and water vapour in the sample, then we have $M_v = r.M_d$.

Applying the formula from question 4 gives $\frac{M_d + M_v}{m_m} = \frac{M_d}{m_d} + \frac{M_v}{m_v}$ or

$\frac{1+r}{m_m} = \frac{1}{m_d} + \frac{r}{m_v}$, or $\frac{m_d}{m_m} = \left\{1 + \frac{r.m_d}{m_v}\right\}\frac{1}{1+r}$

Whence $\hat{T} = \left\{1 + \frac{r.m_d}{m_m}\right\}\left\{\frac{1}{1+r}\right\}T$. This can be evaluated exactly, but since r is small we may approximate it by $\hat{T} \approx \left\{1 + \frac{r.m_d}{m_m}\right\}\{1-r\}.T \approx \left\{1 + r\left(\frac{m_d}{m_m} - 1\right)\right\}T$

In the terms of the question, $r = 0.03$, $T = 298K$ and $\left(\frac{m_d}{m_m} - 1\right) = \left(\frac{29.0}{18.0} - 1\right) \approx \frac{5}{3} - 1 = 0.667$, so that $\hat{T} = \{1 + 0.03 \times 0.667\}298K = 304K$

The virtual temperature of the sample is 31 C.