

Inverse Theory Week 2: Homework Problems

September 29, 2005

This problem sheet is not part of your assessment. It will be useful preparation for the in-class test in week 3.

1. The following 10 pairs of numbers are samples of a 2-element vector random variable \mathbf{x} : (1,2) (4,5) (3,3) (1,3) (2,4) (3,4) (2,2) (2,3) (4,4) (3,5). Calculate the mean value of \mathbf{x} and its covariance matrix, \mathbf{S} .
2. A vector random variable \mathbf{x} has a Gaussian distribution:

$$P(\mathbf{x}) = \frac{1}{(2\pi)^{\frac{n}{2}} |\mathbf{S}|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(\mathbf{x} - \bar{\mathbf{x}})^T \mathbf{S}^{-1}(\mathbf{x} - \bar{\mathbf{x}})\right)$$

which is specified by a mean $\bar{\mathbf{x}}$ and a covariance matrix \mathbf{S} . Sketch the contours of $P(\mathbf{x})$ in the (x_1, x_2) plane for the following two cases. Label your axes (you do not have to label the contours).

(a) $\bar{\mathbf{x}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\mathbf{S} = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$

(b) $\bar{\mathbf{x}} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $\mathbf{S} = \begin{bmatrix} 0.5 & -0.4 \\ -0.4 & 1 \end{bmatrix}$

3. The fundamental constant f is measured by two different experimental groups at the Aardvark National Laboratory and the University of Zorktown. ANL give a value of $f_a = 3.35 \pm 0.04$ in the appropriate physical units, while UZ report a value of $f_z = 3.32 \pm 0.01$. Combine f_a and f_z to give a best combined estimate of f .