Inverse Theory Week 1: In-class problems.

14th January 2005

1 Simultaneous equations.

Solve the following sets of simultaneous equations exactly where that is possible. If a set is underdetermined, consider whether there are no solutions or infinitely many. If there are infinitely many, give two of them, one of which is the solution $\mathbf{x} = \mathbf{K}^T(\mathbf{K}\mathbf{K}^T)^{-1}\mathbf{y}$.

1. $\begin{array}{rcl} 5 & = & 3x_1 + 2x_2 \\ -3 & = & x_1 - 2x_2 \end{array}$
2. $2 = x_1 - 3x_2$
3. $\begin{array}{rcl} 1 & = & 3x_1 + 2x_2 \\ 2 & = & 6x_1 + 4x_2 \end{array}$
4. $\begin{array}{rcl} 1 & = & 3x_1 + 2x_2 \\ 1 & = & 6x_1 + 4x_2 \end{array}$
5. $\begin{array}{rcl} 1 & = & 0x_1 + x_2 - x_3 \\ 1 & = & x_1 + x_2 - 0x_3 \end{array}$

2 Eigen-problems

Find the eigenvalues and eigenvectors of the following matrices. Test the orthogonality of the eigenvectors and comment in the light of the symmetric-ness of the matrix.

1. $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$
2. $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$
3. $\begin{bmatrix} 2 & -2 & -2 \\ -2 & 1 & -1 \\ -2 & -1 & 1 \end{bmatrix}$ (hint: one of the eigenvalues is 2)

What matrix has the eigenvalues 10 and 5 with eigenvectors $(2, 1)$ and $(-1, 2)$ respectively?